Higher order accurate numerical schemes for hyperbolic conservation laws

The system of hyperbolic conservation laws is the first order partial differential equations of the form

$$\frac{\partial \mathbf{u}}{\partial t} + \sum_{\alpha=1}^{d} \frac{\partial \mathbf{f}_\alpha(\mathbf{u})}{\partial x_\alpha} = 0, \qquad (\mathbf{x}, t) \in \Omega \times (0, T], \tag{1}$$

subject to initial data

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}),$$

where $\mathbf{u} = (u_1, u_2, \ldots, u_m) \in \mathbb{R}^m$ are the conserved variables and $\mathbf{f}_\alpha : \mathbb{R}^m \to \mathbb{R}^m$, $\alpha = 1, 2, \ldots, d$ are the Cartesian components of flux. It is well-known that the classical solution of (1) may ceases to exist in finite time, even the initial data is sufficiently smooth. The appearance of shocks, contact discontinuities and rarefaction waves in the solution profile make difficult to devise higher-order accurate numerical schemes because numerical schemes may develop spurious oscillations or sometimes blow up of the solution may occur.

In this talk, we will discuss recently developed Weighted Essentially Non-oscillatory (WENO) schemes of adaptive order for hyperbolic conservation laws, which compute the solution accurately while maintaining the high resolution near the discontinuities in a non-oscillatory manner. Also, I would like to discuss my past and current research works and followed by future research plans.