Ph.D. Thesis Defence

Results on algebraic and arithmetic invariants

By

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Abstract

The study of class groups remains an essential, active and significant area of research in algebraic number theory. The divisibility properties of class numbers provide information for understanding the structure of the class group. We prove that for any odd prime p, if m is a prime power, then the class number of the imaginary quadratic field $\mathbb{Q}(\sqrt{1-2m^p})$ is divisible by p. We also prove a special case of the Iizuka's conjecture.

Weinberger established a link between Euclidean ideals and class groups. He demonstrated in 1972, assuming a generalized Riemann hypothesis (GRH), that for a number field K with $\text{rank}(\mathcal{O}_K^\times) \geq 1$, the number ring $\mathcal{O}_K$, the class group Cl_K is trivial if and only if the number ring $\mathcal{O}_K$ is Euclidean domain. Lenstra extended the concept of Euclidean domains in 1978 by introducing Euclidean ideal classes. By assuming GRH, he proved that Cl_K is cyclic if and only if K has a Euclidean ideal class. We removed the assumption of GRH in low degree number fields.

If L_1/M and L_2/M are finite extensions of complete discrete valuation fields, then under certain conditions, we prove the equality of ramification indices and residue degrees, $e_{L_1/M}e_{L_2/M} = e_{L_1L_2/M}$ and $f_{L_1/M}f_{L_2/M} = f_{L_1L_2/M}$.

Given an elliptic curve $E/\mathbb{Q}$, we briefly discuss several arithmetic and analytic properties of the modular degree of E. We shed some light on Watkin's conjecture.

References

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