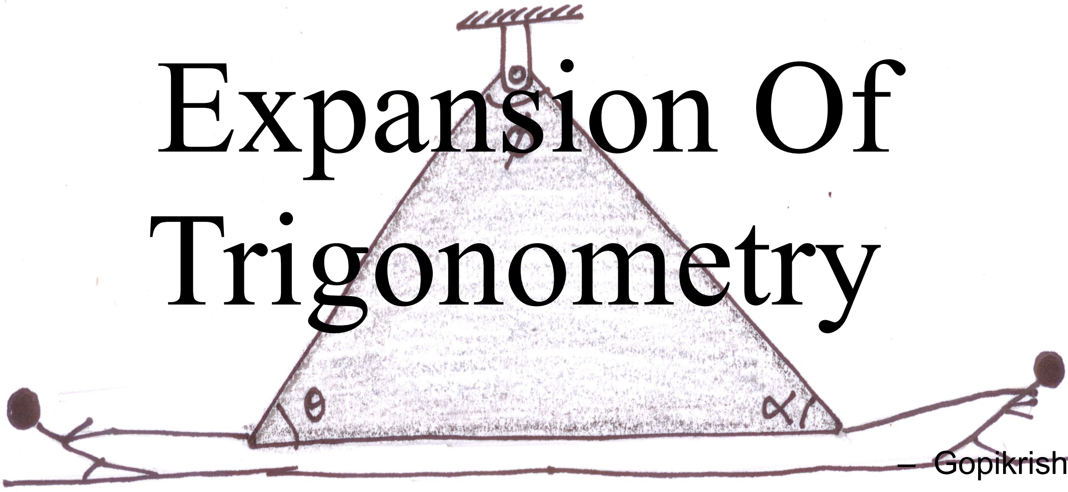
Expansion Of Trigonometry



– Gopikrishnan M.

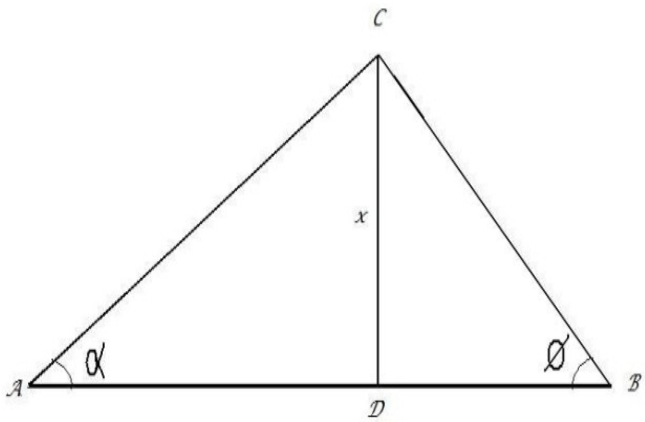
Abstract

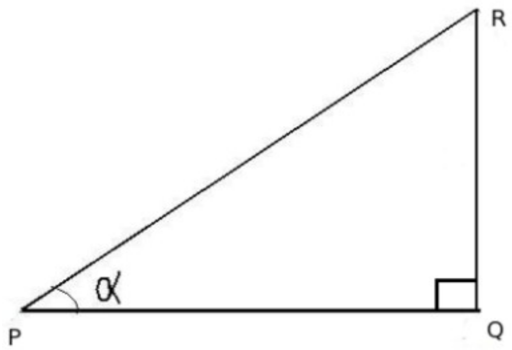
This article deals with the expansion of trigonometry to oblique triangles. It includes some new notations and its connection with our age old trigonometry based on right angled triangles.

Introduction:

In trigonometry, we usually consider only a right angled triangle and define sinα, cosα, tanα (α being one of the two angles) as different ratios of the sides of that triangle. Here we will try to do the same for an oblique triangle. Take any triangle and fix one of the angles say ‘ϕ’ as our standard or base (90° in the case of right angled triangles). Now we can define similar ratios for the other angles of that triangle, say α, like $\sin_{\phi}\alpha$, $\cos_{\phi}\alpha$, $\tan_{\phi}\alpha$, ϕ being our standard angle.

Consider the oblique triangle ABC given below,





We have ∠ABC = ϕ given to us.

Now drop a perpendicular from C to AB.

Let that point of intersection be D.

Note that ΔCDB is a right angled triangle.

Let segment CD = x.

We will now find the trigonometric ratios for ∠CAB = α. Generally, hypotenuse is defined as the side opposite to the right angle. Thus, hypotenuse in triangle ABC will be side opposite to the angle ϕ i.e. side AC.

In a right angled triangle sine of an angle is defined as the ratio of side opposite to that angle to its hypotenuse. In ΔABC, we define,

$$\sin_{\phi}\alpha = \frac{BC}{AC} \qquad (1)$$

Also,

$$\sin\alpha = \frac{x}{AC}, \quad \sin\phi = \frac{x}{BC} \qquad (2)$$

From (1) and (2) we get,

$$\sin_{\phi}\alpha = \frac{\sin\alpha}{\sin\phi} \qquad (*)$$

Now we define

$$\cos_{\phi}\alpha = \frac{AB}{AC} \qquad (3)$$

Also from the diagram,

$$\cos\phi = \frac{BD}{BC},$$

Therefore,

$$BD = BC \times \cos\phi \qquad (4)$$

$$\cos\alpha = \frac{AD}{AC},$$

So,

$$AD = AC \times \cos\alpha \qquad (5)$$

$$AB = AD + BD \qquad (6)$$

From (3) and (6),

$$\cos_\phi \alpha = \frac{AD+BD}{AC}$$
$$\cos_\phi \alpha = \frac{AD}{AC} + \frac{BD}{AC}$$

From (4) and (5),

$$\cos_\phi \alpha = \cos\alpha + \frac{BC}{AC} \times \cos\phi$$
$$\cos_\phi \alpha = \cos\alpha + \frac{\sin\alpha}{\sin\phi} \times \cos\phi$$
$$\cos_\phi \alpha = \cos\alpha + \sin\alpha \times \cot\phi \qquad (**)$$

Similarly, we define,

$$\tan_\phi \alpha = \frac{BC}{AB} = \frac{\sin_\phi \alpha}{\cos_\phi \alpha} \qquad (7)$$
$$\tan_\phi \alpha = \frac{\sin\alpha}{\sin\phi\cos\alpha + \sin\alpha\cos\phi},$$
$$\tan_\phi \alpha = \frac{\sin\alpha}{\sin(\phi + \alpha)} \qquad (***)$$

Thus, we have defined basic trigonometric ratios for any given triangle with one of the angles of the triangle as the standard or base.

They are:

(*) $$\sin_\phi \alpha = \frac{\sin\alpha}{\sin\phi}$$

(**) $$\cos_\phi \alpha = \cos\alpha + \sin\alpha \times \cot\phi$$

(***) $$\tan_\phi \alpha = \frac{\sin\alpha}{\sin(\phi + \alpha)}$$

If we substitute the value of ϕ as 90^0 in the above equations we get back values for the right angled triangle as shown;

$$\sin_{90}\alpha = \frac{\sin\alpha}{\sin 90} = \sin\alpha$$
$$\cos_{90}\alpha = \cos\alpha + \sin\alpha \times \cot 90 = \cos\alpha$$
$$\tan_{90}\alpha = \frac{\sin\alpha}{\sin(90+\alpha)} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

Conclusion:

Thus, the trigonometric ratios can be defined for an angle with respect to any base angle i.e. it can be expanded to triangles other than right angled triangles. I would encourage my dear readers to check out the values of these ratios. I would suggest you to start with 60^0 based trigonometry as it may have great applications due to the fact that it includes the very special equilateral triangle. I want my readers also to come up with different identities that these ratios satisfy, which reduces to our familiar identities when $\phi = 90^0$.